Charles University
Faculty of Mathematics and Physics
Richard Korytár
http://ctcm.kfkl.cz/srfp2020

Summer 2020

24.2.2020

Seminar on problem solving in physics NFPL087, Wed 10:40, F052

Problem sheet 2

Literature: Ashcroft, Mermin: Solid state physics

1 Drude model for conductivity of a metal

Consider a metal in a time-dependent electric field $\mathbf{E}(t) = \mathrm{Re}[\mathbf{E}(\omega)e^{-i\omega t}]$ that induces an electric current density $\mathbf{j}(t)$. In this exercise, we investigate a model for the frequency-dependent conductivity $\sigma(\omega)$ linking the current density to the electric field,

$$\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}(\omega)\,. \tag{1}$$

(a) Consider an electron with momentum $\mathbf{p}(t)$ in the solid. Write the equation that describes the momentum change in time as it is induced by an external time-dependent electric field $\mathbf{E}(t)$.

(b) While traveling through the metal the electron experiences collisions with obstacles such as other electrons, lattice vibrations or crystal imperfections. Model these scattering processes by introducing a friction term that is parameterized by a relaxation time τ.

(c) Assume a monochromatic electric field, $\mathbf{E}(t) = \mathbf{E}(\omega)e^{-i\omega t}$, and compute $\mathbf{p}(\omega)$ as function of $\mathbf{E}(\omega)$.

(d) Suppose that the theory for the momentum relaxation just formulated also describes the average momentum of the electron gas. With this assumption connect the momentum to the electric current density $\mathbf{j}$ and show

$$\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}(\omega)\,, \qquad \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}\,, \qquad \sigma_o = \frac{ne^2\tau}{m}, \tag{2}$$

where m is the mass of an electron and n the material-specific density of charge carriers.

(e) Discuss your results. Begin with the limit of zero frequency and the dependency on τ. What happens in the limit of zero scattering rate? What is implied by the fact that the conductivity in this limit is no longer a real number?

2 Dielectric function of a metal

In this problem we relate the frequency dependent conductivity to the dielectric function. (Ashcroft-Mermin) For simplicity we focus on the limit of very small wavenumbers in which all spatial dependency of $\sigma(\omega)$ and $\epsilon(\omega)$ can be ignored.

(a) Consider Maxwell's equations in materials

$$\nabla \cdot \mathbf{D} = 4\pi\rho\,, \qquad \nabla \cdot \mathbf{B} = 0\,, \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}\,, \qquad \nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}\,. \tag{3}$$

Here ρ and $\mathbf{j}$ denote charge and current density of free charge carriers. Simplify to the case where the localized charges do not contribute significantly to the charge and density response.

(b) Prove that under these conditions the following relation holds

$$-\nabla^2\,\mathbf{E}(\omega) = \frac{\omega^2}{c^2}\,\epsilon(\omega)\,\mathbf{E}(\omega)\,, \tag{4}$$

$$\epsilon(\omega) := 1 + \frac{4\pi i\sigma(\omega)}{\omega}\,, \tag{5}$$

where as usual $\sigma(\omega)$ denotes the zero wave number conductivity. Explain why the prefactor of the first equation has been identified with the dielectric function, second line.

(c) Evaluate the dielectric function in the collisionless limit, $\omega\tau \gg 1$. Show that $\epsilon(\omega)$ simplifies to

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}\,, \tag{6}$$

where ω_p is a material-dependent constant known as the plasma frequency. Compute ω_p and discuss your results.

(d) In the proceeding derivations the wave number dependency of the response functions has been neglected. Discuss the validity of this approximation given that the wave number of light is not vanishing.