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http://ctcm.kfkl.cz/statphys2020

Methods of Statistical Physics
NFPL088, Wed 16:30, zoom/F155KFM

Problem Sheet 4

1 Two-spin molecule
Consider Ising Hamiltonian for a two-atomic molecule (with spins 1/2 - valued $\{+1, -1\}$ on each atom) in an external field B (no interaction outside the molecule).

- **(a)** Find the partition sum of this system.
- **(b)** Find the free energy and the heat capacity c_V of this system assuming $B = 0$.
- **(c)** Decide whether a phase transition is possible, find the temperature of the maximum of c_V.

2 One-dimensional Ising chain
Consider Ising Hamiltonian for a 1D chain of N equivalent spins with only nearest neighbor interaction, no external field, exact (no MFA).

- **(a)** Calculate the partition sum (Hint: prove the recurrence relation $Z_{N+1} = 2Z_N \cosh(J/kT)$). Give the free energy, heat capacity c_V, mean spin value at site n; correlation between different sites m, n.
- **(b)** Decide whether a phase transition is possible, find the temperature of the maximum of c_V, compare to phase transition temperature in MFA.

3 Specific heat of a two-level system
Consider a solid, which has N atoms, and each of them has two energy levels, with eigenenergies Δ and $-\Delta$ as in the Sheet 3, Problem **2**.

- **(a)** Calculate the mean energy, and the heat capacity $c_V = \mathrm{d}E/\mathrm{d}T$. Hint: you can use the results of the Sheet 3, Problem **2**(**a**)
- **(b)** What is the behavior of the heat capacity for $kT \gg \Delta$ and $kT \ll \Delta$?
- **(c)** Sketch a plot of $c_V(T)$ based on these two limits (no computer plot!).
- **(d)** Discussion: Read about the Schottky anomaly and name some materials, where it is observed.

Turn the page!

4 **Specific heat of a glass material**

Consider a solid from the Problem **3** with a modification. The solid is disordered (*e.g.* a glass), so that the values of Δ are random. We assume they are uniformly distributed in the interval $0 \leq \Delta \leq \Delta_0$. Obviously, $2\Delta_0$ is a maximal excitation gap of the atom.

(a) Use the expression for the inner energy from the Problem **3** and perform an average over Δ!

(b) Calculate the heat capacity of the glass and show, that it is linear in T at low temperatures.