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Seminar on problem solving in physics NFPL087, Tue 14:50

Problem Sheet 1

Literature: Kittel's textbook on solids covers all the necessary details on these three problems of ferro/antiferro/ferri-magnetism.

1 Thermodynamics of a classical dipole

A classical dipole moment $\vec{p}$ has a fixed magnitude $p = |\vec{p}|$; the moment directions are confined to a fixed plane ξ. The moment is under influence of an external electric field $\vec{E}$ at a temperature T; the field direction lies in the plane ξ. Consider the limit of low fields and high temperatures and calculate:

(i) the average value $\langle \vec{p} \rangle$ of the dipole moment,

(ii) the corresponding heat capacity (specific heat) C.

2 Ferromagnetism in the mean-field approach

Consider

(i) a single classical spin of magnitude S (a 3D vector of length S)

(ii) a single quantum spin (total spin quantum number $S > 0$ being an integer or half-integer)

in an external magnetic field $\vec{B}$ at a temperature T. Calculate the average value of the magnetic moment $\vec{\mu}$ in the limit of small fields and high temperatures. (Assume that the magnetic moment is proportional to the spin vector with a proportionality constant γ). Discuss the difference between the results of i) and ii).

Now consider a homogeneous lattice occupied by classical magnetic moments (unit vectors) $\vec{e}_R$ interacting with an isotropic nearest-neighbor (exchange) interaction. We will treat this problem in a mean-field approximation and we will consider homogeneous magnetizations only. Thus, the problem effectively reduces to a single moment $\vec{e}$. The mean field acting on each magnetic moment depends on the magnetic moments of its nearest neighbours, $\vec{B}_{mol} = \lambda\langle\vec{e}\rangle$ and favours *parallel* alignment ($\lambda < 0$). Consider the limit of high temperatures and calculate:

(i) the critical temperature(s) T_C,

(ii) the homogeneous susceptibility $\chi(T)$, i.e., the response of the average magnetization to a homogeneous applied field, for temperatures T above T_C.

Discussion: what is the critical exponent in the mean-field approach? Optional: Discuss, what does it mean that critical exponents are *universal* and why are they?

$\boxed{3}$ Critical point of an antiferromagnet in the mean-field approach

Consider a homogeneous lattice occupied by classical magnetic moments (unit vectors) $\vec{e}_R$ interacting with an isotropic nearest-neighbor (exchange) interaction. We will treat this problem in a mean-field approximation and we will consider homogeneous magnetizations only. Thus, the problem effectively reduces to a single moment $\vec{e}$. The mean field acting on each magnetic moment depends on the magnetic moments of its nearest neighbours, $\vec{B}_{mol} = \lambda\langle\vec{e}\rangle$ and favours *antiparallel* alignment ($\lambda < 0$). Consider the limit of high temperatures and calculate:

(i) the critical temperature(s) T_C,

(ii) the homogeneous susceptibility $\chi(T)$, i.e., the response of the average magnetization to a homogeneous applied field, for temperatures T above T_C.

Discussion: Compare the critical behavior of the susceptibitity with the ferromagnetic case.

$\boxed{4}$ Ferrimagnetism

Consider a homogeneous lattice composed of two sublattices A,B occupied by classical magnetic moments $\vec{m}_A$, $\vec{m}_B$ with different magnitudes m_A, m_B within the mean field approximation. The mean (molecular) field acting on each magnetic moment (at position R) depends only on the magnetic moments of its 1st nearest neighbours, $\vec{B}_{mol,A} = \lambda\langle\vec{m}_B\rangle$ and $\vec{B}_{mol,B} = \lambda\langle\vec{m}_A\rangle$, and favours antiparallel alignment ($\lambda < 0$). Consider the limit of high temperatures and calculate:

(i) the critical temperature(s) T_C,

(ii) the homogeneous susceptibility $\chi(T)$, i.e., the response of the average magnetization to a homogeneous applied field, for temperatures T above T_C.

Discussion: an antiferromagnet is a special case of your solution (show!).