

Seminar on problem solving in physics NFPL087, Tue 15:00

Problem Sheet 3

1 Transverse and longitudinal phonons

An infinite one-dimensional periodic chain (lattice parameter a) of mass points (mass m) is located on a strained string (force F). The points can move in a direction

- (a) perpendicular
- (b) parallel

to the string (their shifts are much smaller than a). Derive the equations of motion for the mass points and the phonon dispersion curve. Depict the phonon dispersion curves graphically.

Discuss: Why do certain phonons cost vanishingly small energy? Explain! How is it related to a certain symmetry of the problem?

2 Optical and acoustic phonons

An infinite number of mass points with two different masses can move along a line. The neighboring points are coupled by springs which have the same equilibrium length a and spring constants C , the masses of points form an alternating sequence: $\dots, m, M, m, M, m, M, \dots$, where $m \neq M$. Derive:

- (a) the equations of motion for the mass points,
- (b) the phonon dispersion curve of the two branches, *optical* and *acoustic*,
- (c) the velocity of sound.

Depict the phonon dispersion graphically for wave vectors in the 1st Brillouin zone.

Discuss the long wavelength behavior of the phonon energies: Is the phonon energy (of each branch) finite or vanishing? What is the role of symmetry? In order to answer this question, you only need to imagine the relative motion of the two atoms in the unit cell.

3 Phonon spectrum in presence of an impurity (mass defect)

Consider an infinite one-dimensional chain of mass points. The points can move along a line; the neighboring points are coupled by identical springs (spring constant C). The mass of one of the points is M , the mass of all other points is m . Derive:

- (a) the equations of motion for the mass points,
- (b) a condition for existence of a localized phonon mode,
- (c) the frequency ω_L of the localized phonon mode,

- (d) a relation between ω_L and the upper frequency ω_0 of the phonon spectrum of an infinite 1D chain of mass points m coupled by the same springs ($\omega_0 = 2\sqrt{C/m}$).

Literature: Mihály László, *Solid state physics: Problems and solutions*

4 A 2D lattice

Consider an infinite square two-dimensional lattice of mass points with nearest-neighbor distance a . The points can move in one direction σ only; the neighboring points are coupled by identical springs (spring constant C). The mass of all points is m . Derive:

- (a) the equations of motion for the mass points,
- (b) the phonon dispersion curves.

Depict the phonon dispersion graphically, decide about the boundaries of the 1. BZ.