Correlation functions

Reminder from the course
“Methods of Statistical Physics”

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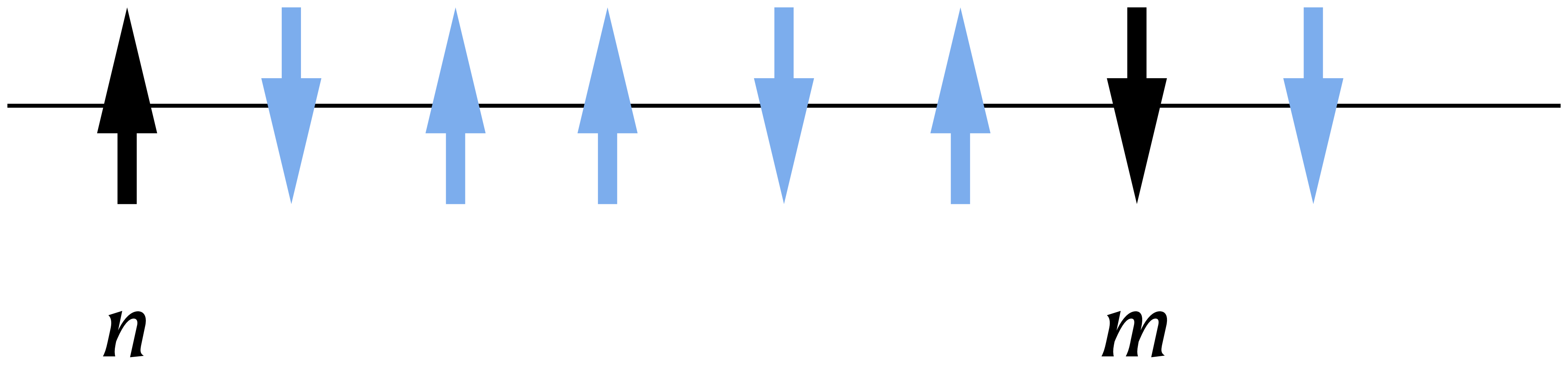
Example 1: Ising model



- Spin - spin correlation function $\langle S_n S_m \rangle$
  - Conditional probability
  - Ornstein-Zernike behavior in the disordered phase

$$\langle s_m s_n \rangle (T) \sim \frac{1}{d_{mn}} \exp\left[-\frac{d_{mn}}{\xi(T)} \right] , \tag{44}$$

where the $d_{mn} = |\mathbf{T}_m - \mathbf{T}_n|$ denotes the intersite distance.

Correlation function describes linear response

- The spin-spin correlation function is proportional to the susceptibility

$$\langle s_m s_n \rangle (T) = k_B T \, \chi_{mn}(T)$$

correlation function

"fluctuation"

linear response to
the magnetic field

- This is one form of the fluctuation-dissipation theorem

Example 2: Density response

Electron system in a perturbing time-dependent potential

$$\delta\varrho(\mathbf{r}, t) = \int\int_{-\infty}^{t} \chi_0(\mathbf{r}, \mathbf{r}', t, t')\, \delta V(\mathbf{r}', t')\, \mathrm{d}^3\mathbf{r}'\, \mathrm{d}t$$

Change of the density

Perturbing potential

Generalized susceptibility
of the linear response

- Most general form of linear response, respecting causality
- Response functions $\chi_{AB}(\omega)$, here *A*=*B* is the density operator ***n*(r)**

Generalized susc. and correlation functions

- Autocorrelation function

$$\Gamma_A(t) = \frac{1}{2}\langle A(t)A + AA(t)\rangle$$

- Fluctuation-dissipation theorem

$$\tilde{\Gamma}_A(\omega) = -\coth\left(\frac{\beta\omega}{2}\right)\chi^{(2)}_{AA}(\omega)$$

Correlation/fluctuation

Linear response of an observable

Seminar on problem solving in physics NFPL087, Tue 15:00

Problem Sheet 6

TOPIC: Autocorrelation functions and spectroscopy

*Classical physics only

1 Harmonic oscillator

Calculate the autocorrelation functions for a classical linear harmonic oscillator in thermodynamic equilibrium at temperature T. The Hamiltonian of the system is $H(p,q) = p^2/(2M) + (M\Omega^2/2)q^2$, where M denotes the mass and Ω denotes the frequency.

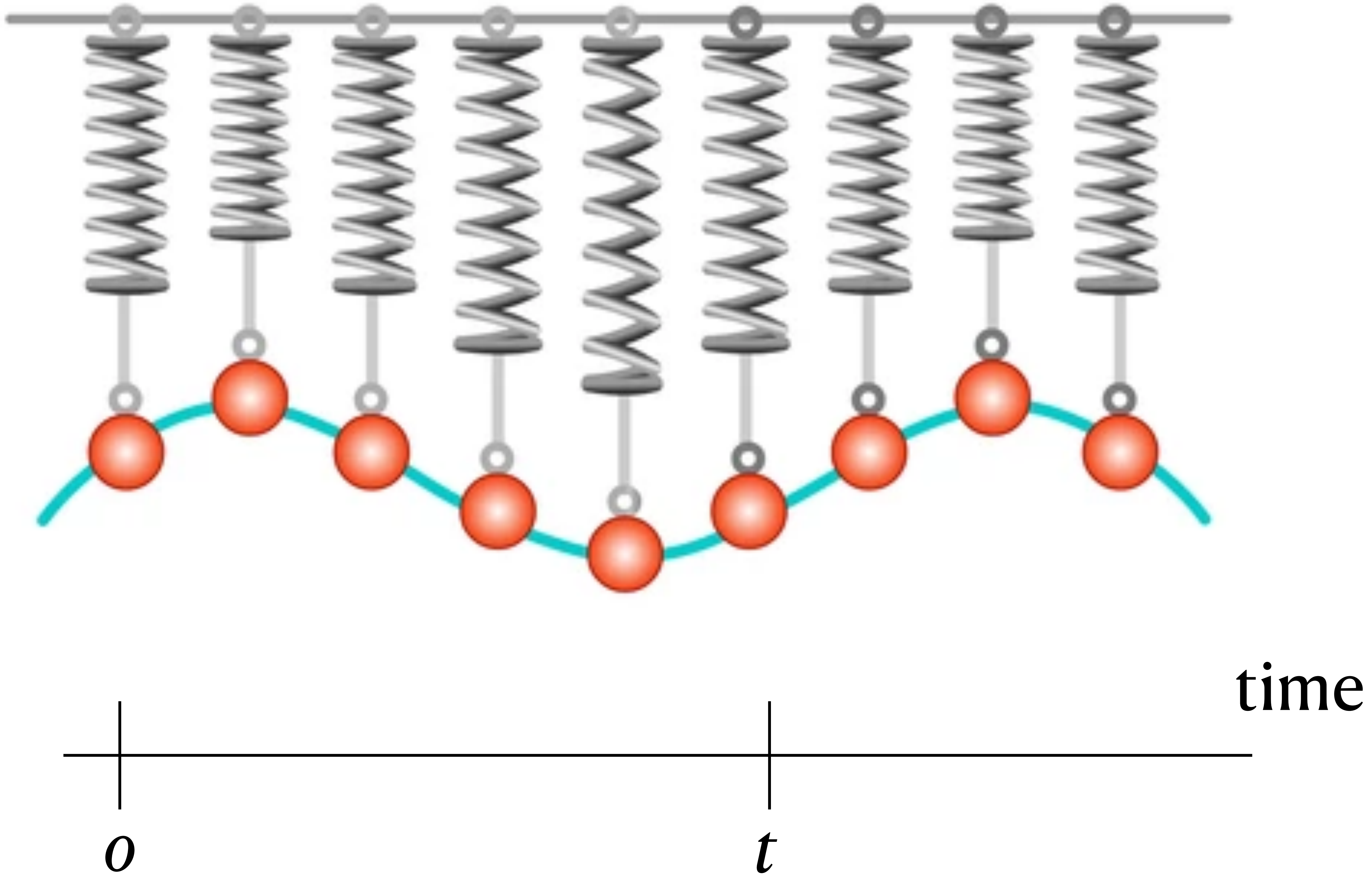
Autocorrelation functions as conditional probabilities

- Momentum autocorrelation function

$$\Gamma_{pp}(t) := \langle p(t)p(0) \rangle$$

- Position autocorrelation function

$$\Gamma_{qq}(t) := \langle q(t)q(0) \rangle$$



$\Gamma_{qq}(t) := \langle q(t)q(0)\rangle$ is large and positive if the probability of the oscillator to return to the original position at $t=0$ is large