

Methods of Statistical Physics
NFPL088, Tue 10:30, F052 (KFKL)

Problem Sheet 1

1 Thermodynamics of a classical magnetic moment in the magnetic field

Consider a single classical magnetic moment $\boldsymbol{\mu}$ in an external magnetic field \mathbf{B} at a temperature T .

- (a) Calculate the average value of $\boldsymbol{\mu}$ in the limit of small fields and high temperatures.
- (b) Calculate the magnetic susceptibility.

2 Thermodynamics of a quantum spin in the magnetic field

Repeat the previous problem with a quantum spin $\hat{\mathbf{S}}$ with a fixed square, $\hat{\mathbf{S}}^2 = S(S+1)$ (e.g. an electron or certain atoms). The Hamiltonian describing the free spin in a field is $\hat{H} = g\mu_B \mathbf{B} \cdot \hat{\mathbf{S}}$.

3 Classical gas in a gravitational field

Let's assume N classical point masses moving along a half-line $x > 0$ subject to a constant gravitational force, mg . The Hamilton's function is

$$H(\{x_i\}_{i=1}^N, \{p_i\}_{i=1}^N) = \sum_{i=1}^N \left(\frac{p_i^2}{2m} + cx_i \right), \quad (1)$$

where $x_i > 0$ are the coordinates, p_i the momenta and m is the mass ($c = mg$). For a classical canonical distribution at temperature T , derive

- (a) the partition sum;
- (b) the average particle density $\langle \rho(x) \rangle$, where $\rho(x) = \sum_i \delta(x - x_i)$. Verify, that your result gives the correct particle number, $\int dx \langle \rho(x) \rangle = N$. Qualitatively discuss the temperature dependence.
- (c) Calculate the internal energy. (Hint: it's not necessary to compute another integral.) Discuss the relation with equipartition theorem.

4 Density of states of free particles: boundary effects

Consider free particles with a parabolic dispersion $\varepsilon_{\mathbf{p}} = \mathbf{p}^2/2m$ in a three-dimensional box of length L . Calculate the density of states using

- (a) periodic boundary conditions

$$\psi(x + L, y, z) = \psi(x, y, z)$$

(and similar conditions in the y and z directions);

(b) hard-wall boundary conditions

$$\psi(0, y, z) = \psi(L, y, z) = 0$$

(and similarly in the y and z directions).

Show, that the single-particle ground-state $\psi^{(E=0)}(x, y, z)$ differs between cases (a) and (b), although the densities of states are equal.

5 Density of states of free particles: two dimensions

Show, that the density of states in two dimensions is $N(\epsilon) = \Theta(\epsilon) \times L^2/\delta E$, where $\Theta(\epsilon)$ is the Heaviside function and δE is the difference between the first-excited and ground-state energies.

6 Density of states of free particles: three dimensions

Derive the formula for the density of states in three dimensions for a free “particle-in-a-box”.