Charles University
Faculty of Mathematics and Physics
Richard Korytár

Winter 2021

4.12.2021

http://ctcm.kfkl.cz/statphys2021

Methods of Statistical Physics
NFPL088, Tue 11:30, F052 (KFKL)

Problem Sheet 5

We will continue with the problems from last time:

1 **Two-spin correlations in the mean-field approximation (MFA)**
In the MFA to the Ising model, calculate $\langle s_n s_m \rangle_0$ (directly perform the phase space average).

2 **Model of an atom in a radiation field**
We consider an atom which can be excited, either thermally or by the external field. For simplicity, we focus on the ground state and the first excited state only. Naturally, it is convenient to use Pauli matrices $\hat{\sigma}_i$, acting on the two atomic states. The Hamiltonian of the atom reads $\hat{H}_\mathrm{A} = \Delta\hat{\sigma}_z$. The Hamiltonian of the field, capable at inducing transitions, is $\hat{H}_\mathrm{F} = \varepsilon\hat{\sigma}_x$. Study the linear response to $\hat{H}_\mathrm{F}$ in the following steps:

- **(a)** To familiarize yourself with the model, give the spectrum of $\hat{H}_\mathrm{A}$, the energetic gap and the probability of finding the atom in an excited state at a given temperature.
- **(b)** Compute the susceptibility $\kappa_{AB}(T)$ with $\hat{A} = -\hat{B} = \hat{\sigma}_x$ following the definition given in the lecture [Eq. (41) therein]. You can think that the operator σ_x measures the dipole moment.
- **(c)** For $\Delta \neq 0$, evaluate the limits $T = 0$ and ∞ of the susceptibility. Then, evaluate the limit $\Delta = 0$ for any T. If the ground state is degenerate, the zero-temperature susceptibility should diverge.
- **(d)** Compute the susceptibility $\kappa_{AB}(T)$ with $\hat{A} = \hat{\sigma}_x, \hat{B} = \hat{\sigma}_z$. Discuss the result: what does it imply?

Further reading: Quantum optics often employs the so called Jaynes–Cummings model, where $\hat{H}_\mathrm{A}$ is used to model an atom. The field is quantized, unlike in this problem, where it is classical and static.

3 **Curie-like temperature behavior**
Using the definition of the susceptibility [Eq. (41) from the lecture] show, that the susceptibility of a non-degenerate system is finite at low temperatures, and diverges if the system has a ground-state degeneracy. Discuss the examples: magnetic susceptibility of a Fermi gas and of a free magnetic moment.