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Winter 2022

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Methods of Statistical Physics
NFPL088, Tue 17:20, F155 (KFM)

Problem Sheet 2

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1 Fluctuation-dissipation relation: classics

Prove the relation

$$(\Delta H)^2(T) = k_{\mathrm{B}} T^2 C(T) \tag{1}$$

in classical statistics, using the definitions of $F(T), S(T)$ and $Z(T)$ given in the lecture.

2 Fluctuation-dissipation relation: quantum statistics

Prove the relation

$$(\Delta H)^2(T) = k_{\mathrm{B}} T^2 C(T) \tag{2}$$

in quantum statistics, using the definitions of $F(T), S(T)$ and $Z(T)$ given in the lecture.

3 Response function

Proove the relationship given in the lecture:

$$-Z^{-1}(T,0)\times \operatorname{Tr}\left[A\exp(-\beta H_0)\int_0^\beta \exp(\alpha H_0)\, B\exp(-\alpha H_0)\,\mathrm{d}\alpha\right] = \sum_{mn} A_{mn}B_{nm}\,\frac{w_m(T)-w_n(T)}{E_m-E_n}$$

Hint: evaluate the operator products in the orthonormal basis.

4 Statistical mechanics of neutron beams

Suppose that a neutron beam is prepared in the following way: half of the neutrons have their spin aligned with the (positive) x-axis and the other half along the z-axis.

(a) Define the appropriate density operator $\hat{\rho}$ for the beam. Verify that $\hat{\rho}$ satisfies the three basic properties: hermicity, positivity and unitarity. Does the beam represent a pure state?

(b) Calculate the average spin vector $\langle \mathbf{s}\rangle = \frac{1}{2}\hbar\langle\hat{\boldsymbol{\sigma}}\rangle$ and the corresponding variances $\langle \hat{s}_i^2\rangle - \langle s_i\rangle^2$, $i = x, y, z$. Discuss your results.

(c) Calculate the eigenvalues P_+, P_- of the density operator and the so called degree of polarization defined through

$$\Pi := \frac{P_+ - P_-}{P_+ + P_-}. \tag{3}$$

How do you interpret your result?