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http://ctcm.kfkl.cz/srfp2023

Seminar on problem solving in physics
NFPL087, Wed 10:40, F2

Problem Sheet 3

$\boxed{1}$ **Optical properties of systems with polaritons**
As a continuation of Problem $\boxed{\mathbf{3}}$ from Sheet 2, derive the dielectric constant in a material with polaritons. Determine the frequency range in which the dielectric constant becomes negative and interpret your result. Give an example of a measurement that would indicate the polaritonic coupling.

$\boxed{2}$ **Relation of the complex conductivity to dissipation**
Consider an electric field in a medium. The dissipated (absorbed) power $P(t)$ of the field equals the work done by the field on the charges per time, $P(t) = \mathbf{j}(t) \cdot \mathbf{E}(t)$. Show, that in case of a harmonic field the absorbed power is proportional to the real part of the complex conductivity, in the following steps:

(a) In the harmonic case,

$$\mathbf{j}(t) = \mathrm{Re}\{\mathbf{j}(\omega)e^{-\mathrm{i}\omega t}\} \tag{1a}$$
$$\mathbf{E}(t) = \mathrm{Re}\{\mathbf{E}(\omega)e^{-\mathrm{i}\omega t}\}. \tag{1b}$$

The power $P(t)$ will be oscillatory, but we are interested only in the “DC” component, P_{DC}, that we obtain by integrating $P(t)$ over the oscillation period $T = 2\pi/\omega$:

$$P_{\mathrm{DC}} = \frac{\omega}{2\pi}\int_0^T P(t)\mathrm{d}t. \tag{2}$$

Express the integral using the harmonic expressions, Eq. 1.

(b) Employ the linear response relation for the complex amplitudes $\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}(\omega)$ and evaluate the above integral. Hint: The integral has the form $\int \mathrm{Re}\{a\}\,\mathrm{Re}\{b\}\mathrm{d}t$. Use the following trick: $\mathrm{Re}\{a\}\,\mathrm{Re}\{b\} = \frac{1}{2}\,\mathrm{Re}\{ab + a^*b\}$. The integrand has a clearly identifiable DC component.

(c) Verify the result $P_{\mathrm{DC}} = \frac{1}{2}\,\mathrm{Re}\{\sigma(\omega)\}|\mathbf{E}(\omega)|^2$.

(d) Recall the Drude model; what can you say about dissipation in the low and high frequency limits? What happens in the limit $\tau \to \infty$?

$\boxed{3}$ **Transverse and longitudinal phonons**
An infinite one-dimensional periodic chain (lattice parameter a) of mass points (mass m) is located on a strained string (force F). The points can move in a direction

(a) perpendicular

(b) parallel

to the string (their shifts are much smaller then a). Derive the equations of motion for the mass points and the phonon dispersion curve. Depict the phonon dispersion curves graphically.

Discuss: Why do certain phonons cost vanishingly small energy? Explain! How is it related to a certain symmetry of the problem?

4 Optical and acoustic phonons

An infinite number of mass points with two different masses can move along a line. The neighboring points are coupled by springs which have the same equilibrium length a and spring constants C, the masses of points form an alternating sequence: $\ldots, m, M, m, M, m, M, \ldots$, where $m \neq M$. Derive:

(a) the equations of motion for the mass points,

(b) the phonon dispersion curve of the two branches, *optical* and *acoustic*,

(c) the velocity of sound.

Depict the phonon dispersion graphically for wave vectors in the 1st Brillouin zone.

Discuss the long wavelength behavior of the phonon energies: Is the phonon energy (of each branch) finite or vanishing? What is the role of symmetry? In order to answer this question, you only need to imagine the relative motion of the two atoms in the unit cell.

5 Phonon spectrum in presence of an impurity (mass defect)

Consider an infinite one-dimensional chain of mass points. The points can move along a line; the neighboring points are coupled by identical springs (spring constant C). The mass of one of the points is M, the mass of all other points is m. Derive:

(a) the equations of motion for the mass points,

(b) a condition for existence of a localized phonon mode,

(c) the frequency ω_{L} of the localized phonon mode,

(d) a relation between ω_{L} and the upper frequency ω_0 of the phonon spectrum of an infinite 1D chain of mass points m coupled by the same springs ($\omega_0 = 2\sqrt{C/m}$).
Literature: Mihály László, *Solid state physics: Problems and solutions*

6 A 2D lattice

Consider an infinite square two-dimensional lattice of mass points with nearest-neighbor distance a. The points can move in one direction σ only; the neighboring points are coupled by identical springs (spring constant C). The mass of all points is m. Derive:

(a) the equations of motion for the mass points,

(b) the phonon dispersion curves.

Depict the phonon dispersion graphically, decide about the boundaries of the 1. BZ.